output less the sector's final demand. Neither of these approaches are satisfactory, because the output vector is not preserved. If the current level of output is used as the direct contribution, then the input-output model will necessarily overestimate the total contribution because of the multiplier effect (Equation 14). On the other hand, if the direct contribution of a sector is defined as output net of final demand, then the input-output model will tend to underestimate the total contribution because of the inter-dependencies inherent within the input-output framework.

The contribution of an existing single sector of the economy can be calculated with an adjustment factor that preserves the output values in the transactions table. For the single-sector case, the adjustment factor is the reciprocal of the sector's multiplier.

$$Adjustment\ Factor = Multiplier^{-1} \tag{15}$$

The product of the adjustment factor and the level of output is defined as the direct contribution of the sector to the economy.

$$Contribution_{direct} = Output_{total} \times Multiplier^{-1} \tag{16}$$

In the previous example, a multiplier of 1.69 was calculated for the manufacturing sector. The direct contributions made by the manufacturing sector are obtained simply as the product of total manufacturing output and the reciprocal of its multiplier: $350 \times {}^{1}/_{1.69} = 207$, a value which fully preserves the transaction table's output value for manufacturing. The product of the A matrix and the column vector containing the newly calculated manufacturing direct effects is a new column vector or the preserved output vector:

$$\begin{bmatrix} 1.82 & 0.08 & 0.01 \\ 0.45 & 1.44 & 0.18 \\ 0.19 & 0.17 & 1.33 \end{bmatrix} \times \begin{bmatrix} 0 \\ 207 \\ 0 \end{bmatrix} = \begin{bmatrix} 17 \\ 298 \\ 35 \end{bmatrix} \tag{17}$$

In the example, total output in MFG is preserved; the sum of the elements in the new vector is 350 units. The primary contribution of MFG is 298, which is the direct contribution equal of 207 units, and an indirect contribution equal to 91 units (298-207) Secondary contributions from AG (17) and SVC (35) total 52 units. The total of the column vector is the initial level (350) of manufacturing output: 298 of direct manufacturing plus 52 of agriculture and service (Table 2).

TABLE 2. Contribution analysis of manufacturing sector in hypothetical economy.

Manufacturing (primary contribution)	298
Agriculture (secondary contribution)	17
Services (secondary contribution)	35
Total Manufacturing Output	**350**

Multiple Sector Contribution Analysis

In the previous section, we explained how to obtain primary and secondary contributions of a single economic sector. In this section, we expand the procedure to two or more sectors. Understanding the contribution of multiple sectors is often warranted because of the similarity of disaggregated activities. For example, within the broad category of trade, retail and wholesale activities are often disaggregated and tabulated separately. Another example is agriculture and forestry, which are often disaggregated into multiple sectors to represent all of their production and processing activities.

Yet disaggregated sectors that share some similarity within their larger grouping are often major components of the backward linkages of other sectors because they each make substantial transactions within the larger grouping. For example: production agriculture and food processing are both part of agriculture, but have obvious linkages; timber production and furniture manufacturing is another example. Calculation of the contribution of each sector with individual sector multipliers does not adequately account for inter-industry dependencies, and the total output vector will not be preserved. For example, consider the contribution analysis of the example economy simultaneously using AG, MFG, and SVC, which have total output multipliers of 2.46, 1.69, and 1.51, respectively. The single-sector direct contribution value for AG is 73.2 units (180/2.46), 207.1 units (350/1.69) for MFG, and 138.2 units (210/1.52) for SVC. Compared to the totals in Table 1, AG is underestimated (151 vs 180), and MFG (356 vs 350) and SVC (233 vs 210) are overestimated:

$$\begin{bmatrix} 1.82 & 0.08 & 0.01 \\ 0.45 & 1.44 & 0.18 \\ 0.19 & 0.17 & 1.33 \end{bmatrix} \times \begin{bmatrix} 73.2 \\ 207.1 \\ 138.2 \end{bmatrix} = \begin{bmatrix} 151 \\ 356 \\ 233 \end{bmatrix} \tag{18}$$

An alternative that will preserve the output vector is to first aggregate the multiple sectors into a single sector, and then apply the methodology described in the previous section. Unfortunately, aggregating multiple sectors into a single sector is known to create aggregation bias, unless the sectors have identical interindustry structures (Miller and Blair, 2009).

The solution to the multiple sector problem is obtained such that the primary and secondary contributions of all the sectors of interest (e.g., logging and furniture manufacturing) are considered simultaneously, and total output for each sector is preserved. Let X' be a non-zero column vector of output values for the sectors of interest (i.e., an arbitrary subset of the X matrix). Let [i-a] be a subset of the [I-A] matrix that corresponds to the sectors of interest. The product of the [i-a] matrix and X' is a resulting column vector Y', whose elements represent primary direct contributions for each sector of interest (i.e., [i-a] X' = Y'). In the example the primary direct contributions for the three sectors are 90.3 for AG, 196.9 for MFG, and 117.3 for SVC:

$$\begin{bmatrix} 0.56 & -0.03 & 0.00 \\ -0.17 & 0.71 & -0.10 \\ -0.06 & -0.09 & 0.76 \end{bmatrix} \times \begin{bmatrix} 180 \\ 350 \\ 210 \end{bmatrix} = \begin{bmatrix} 90.3 \\ 196.9 \\ 117.3 \end{bmatrix} \tag{19}$$

TABLE 3. Total industrial output of the wood fiber system for a county economy according to IMPLAN Study Area Data.

Industry Code	Description	Total Output
[16]	Commercial logging	$57,764,028
[95]	Sawmills and wood preservation	$33,367,230
[105]	Paper mills	$314,059,904

TABLE 4. Wood fiber system multipliers for a county economy.

Description	Industry Code	[16]	[95]	[105]
Commercial logging	[16]	1.299989	0.418333	0.081726
Sawmills and wood preservation	[95]	0.002965	1.046941	0.011549
Paper mills	[105]	0.000029	0.000042	1.001880

Each element of the Y' vector represents each sector's primary contribution to itself (e.g., 90.3 for AG). The secondary contribution of each sector is equal to the total output of each sector minus its primary contribution (e.g., the secondary contribution for AG is 59.7 or 150 - 90.3).

Multiple Sectors Contribution Analysis and IMPLAN

A popular software used for input-output modeling by economic practitioners and policy analysts is IMPLAN. The modeling system produced by IMPLAN LLC has made providing economic impact data and modeling to governments, universities, and public and private sector organizations for assessing the economic impacts of project decisions in all industry sectors (MIG, 2000).

The popularity of IMPLAN can be attributed to both ease of model development, and geographic flexibility. While the use of IMPLAN to quantify economic effects has been well documented, there is very little information about how it can be used to assess the economic contributions of an existing industry or sector of the economy. IMPLAN (2013) provides a methodology for estimating the contribution of single and multiple industries. This single industry approach is similar to that which we previously described, however his extension to the multi-industry analysis involved extensive model customization.

In this section we demonstrate, using IMPLAN, how the contributions of multiple sectors are calculated without any changes to the model's structural accounting matrix. The analysis demonstrated focuses on the wood fiber industry which consists of three interrelated sectors: commercial logging, sawmill manufacturing, and paper product manufacturing (Table 3).

Enterprises that make up each of these sectors are components of the backward linkages for enterprises in other sectors of wood fiber industry. Therefore the goal is to calculate the primary direct contributions of each sector simultaneously, and preserve the output values. The total output values from these three sectors form the Y' matrix described previously.

The next step is to calculate the [i-a] matrix based on the sectors of interest. IMPLAN's Detailed Multipliers Explorer is used to obtain Type SAM output multipliers (Table 4).

The columns contain Industry Type SAM output multipliers for each sector. For example, the Sawmills and wood preservation sector has a direct Type SAM multiplier of 1.046941, and indirect multipliers of 0.418333 and 0.000042 for Commercial logging and Paper mills, respectively. Collectively these output multipliers form a miniature Leontief Inverse, $[i\text{-}a]^{-1}$, which inverted becomes the [i-a] matrix.

$$(\boldsymbol{I}-\boldsymbol{A})^{-1}=\begin{bmatrix}1.299989 & 0.418333 & 0.081726\\ 0.002965 & 1.046941 & 0.011549\\ 0.000029 & 0.000042 & 1.001880\end{bmatrix} \tag{20}$$

$$(\boldsymbol{I}-\boldsymbol{A})=\begin{bmatrix}0.769940 & -0.307648 & -0.059260\\ -0.002180 & 0.956035 & -0.010843\\ -0.000022 & -0.000031 & 0.998126\end{bmatrix} \tag{21}$$

The next step is to calculate the vector of primary direct contributions (X'):

$$\begin{bmatrix}0.769940 & -0.307648 & -0.059260\\ -0.002180 & 0.956035 & -0.010843\\ -0.000022 & -0.000031 & 0.998126\end{bmatrix}\times\begin{bmatrix}57{,}764{,}028\\ 33{,}367{,}230\\ 314{,}059{,}904\end{bmatrix}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \tag{22}$$

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}15{,}598{,}404\\ 23{,}369{,}056\\ 313{,}468{,}939\end{bmatrix}\begin{matrix}\leftarrow \textit{Commercial logging}\\ \leftarrow \textit{Sawmills and wood preservation}\\ \leftarrow \textit{Paper mills}\end{matrix} \tag{23}$$

The final step is to enter these values as Industry Sales in IMPLAN's Setup Activities dialog (Figure 1). After shocking the model, totals for each sector (Table 5) closely approximate IMPLAN Study Area Data (Table 3). Thus, the total effect values for the sectors of interest reflect the size of those sectors within the economy.

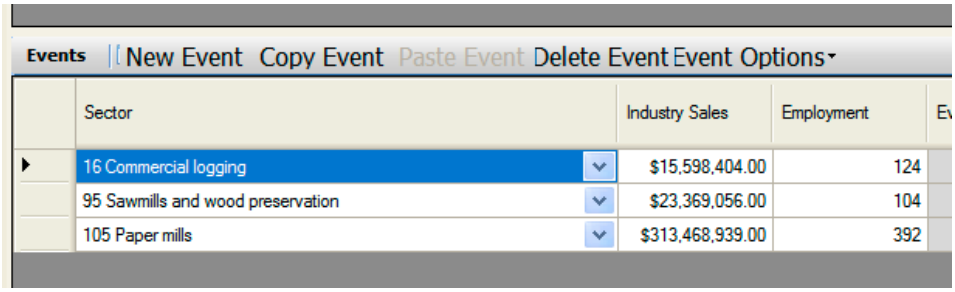

Figure 1. IMPLAN's Setup Activities dialog box showing the sectors of interest and associated industry sales values.